# Atomism and the Reasoning by a Non-Classical Logic

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**Abstract:** Often, in the original scientific writings, a double negated statement (DNS) is not equivalent to his corresponding positive one; that means the inferring law $\neg\neg A \rightarrow A$ does not apply. Recent studies recognized in the failure of this logical law the borderline between classical and non-classical logics. Original writings by classical chemists dealing with the problem of atomism are particularly characterized by the occurrences of DNSs. An historical case, Avogadro's contribution to atomism (*i.e.* the well-known hypothesis about the constitution of gases), is here analyzed in such terms. It turns out that, in order to support his ideas, Avogadro suggested several *ad absurdum* proofs, indeed a way of reasoning typically linked to the use of DNSs.

**Keywords:** *atomism, logic, Avogadro*

## Introduction

More than a century ago, classical logic was formalized in mathematical terms. Next, some 'deviant' mathematical logics were born whose status was for a long time dubious. Finally, some decades ago mathematical logic was recognized not to be unique, as generally believed. At a formal level what discriminates the classical logic from the non-classical ones (say absolute, positive, minimal and intuitionistic logics), is the axiom $\neg\neg A \rightarrow A$ (read: 'not-not-*A* implies *A*', where *A* is any statement), which holds true for classical logic, whilst it fails for non-classical ones.[1] In other words, the basic difference between classical and non-classical logics may be confined to the law of double negation: if classical logic applies, a double negated sentence implies its related positive, otherwise we have what we call a DNS, *i.e.* a double negated sentence, which (and that is the fundamental point) is not equivalent to the related positive one.

By an historical investigation one may recognize the theoretical origin of these DNSs in Leibniz' principle of sufficient reason; even the sentence of this principle is a DNS: "… rien <u>n</u>'est <u>sans</u> raison, ou que toute vérité a sa

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preuve *a priori* tirée de la notions des termes, quoyqu'il ne soit pas toujours en notre pouvoir de parvenir à cette analyse";[2] where Leibniz first states the principle as a $\neg\neg A$, then he tries to draw the corresponding $A$, yet he has to recognize it as impossible in most cases.

The failure of the double negation law ($\neg\neg A \rightarrow A$) may be plainly expressed by the use of double negation of a single statement, really a common linguistic scheme – 'litote' – expressing the thought of a layman; whereas, for instance, the failure of the excluded middle law – in formal terms $A \vee \neg A$ (read: $A$ or not $A$) – has to be deliberately emphasized by an author through an explicit argument. Thus, the occurrence of such DNSs in a scientific text may point out that the writer is not applying the double negation law since he is arguing by a non-classical logic. A linguistic analysis of the original writings of a scientist can reveal his style of argumentation different from classical logic. Of course, we do not claim that people deliberately argued by means of a non-classical logic before it was recognized and properly formalized. No doubt each of these scientists overtly committed himself to classical logic (it was simply 'the logic' for what was manageable by the scientific reason of the time, *i.e.* syllogistic and later propositional sentences). Rather we suggest that in the past some scientists were inclined by some uncommon situations – we will define in the following – to search for new ways of argumentation, and therefore they made an *ante litteram* use of non-classical logic.

Our investigation of ways of reasoning begins with the linguistic analysis of writings in order to mark all recurring DNSs. Here we apply this research method to writings pertaining to 'hard' sciences. (All double negated sentences we will report in the following are DNSs, *i.e.* double negated sentences not reducible to the related positive ones.) In science a DNS generally points out a problem; *i.e.,* there is no sufficient experimental evidence to assert $A$, while, on the other hand, there is some direct and/or indirect evidences – often an *ad absurdum* argument[3] –, that 'it is <u>not</u> possible that $A$ is <u>not</u> true'. At the same time a DNS offers a methodological principle saying that the subsequent theory should not contain the assertion $A$; for instance, the <u>im</u>possibility of a motion <u>without</u> an end prevents the theory from stating a perpetual motion.

In the following, we begin by listing some instances of DNSs in science. Then we will focus our attention upon chemistry, in particular on the texts pertaining to the early historical phase of chemical atomism, which was developed in the first half of 19th century. Finally, we analyze in detail the case of Amedeo Avogadro, in order to get a deeper insight in the way he first formulated the well-known hypothesis about the constitution of gases; actually he made use of several DNSs and *ad absurdum* proofs. As a result, we show that arguing by DNSs was a characteristic feature of classical chemistry.

## 1. Double negated statements in science

In several scientific theories one may find many instances of DNSs in the books written by the respective founders of these theories. For the sake of brevity, we quote some instances only.

In mathematics an example is in L. Carnot's writing which re-founds infinitesimal analysis in an operative way: "The infinitesimals are not chimerical (=not not real) beings".[4] While there is no direct evidence for the existence of infinitesimals, there is evidence for the absurdity of not-existence. A more striking example is offered by N.I. Lobachevski when he discovered non-Euclidean geometry: "1° Rien ne s'oppose à admettre que la somme des angles d'un triangle rectiligne soit moindre que deux angles droits".[5]

In physics many authors of mechanics, thermodynamics and electricity stressed the same basic principle, first announced by Stevin in the following way: "A motion without an end [commonly said a *perpetuum mobile*] is absurd".[6] Since the ignorance of friction during the motion does not allow to determine the final time of the motion, one cannot strictly infer the corresponding sentence: "Any motion has an end". In mechanics, L. Carnot's inertia statement is the following one: "Once a body is put at rest, it cannot move by itself [= without other bodies]; once it is in motion, it can change by itself neither its direction nor its velocity".[7] Let us remark that Newton had stated the same idea as an abstract principle, by saying that a body "perseveres" (or "continues") its motion, *i.e.* by making use of an idealistic concept just in order to change a DNS into an alternative form. In L. Carnot's mechanics the main idea is the following one: "Il y a donc [...] dans toute percussion ou communication de mouvement [...] une quantité qui n'est point altérée par le choc";[8] *i.e.* an invariant (= something not unequal) of the motion; which does not mean at all something equal to itself. In thermodynamics Joule concluded his experiments by the statement: "It is patently absurd to guess that forces God gave to matter may be destroyed by man's action [...] or even created";[9] that constitutes a new linguistic way to state a conservation law by means of DNS. In thermodynamics, again, the main problem is the "equivalence of heat and work". That does not means at all that heat is (equal to) work, because we know that only part of the heat can be converted into work (an engine capable to convert all heat into work cannot exist). On the other hand, "heat is not work" is a false statement. Rather "it is not true that heat is not work" turns out to be the appropriate sentence.[10]

In the 1930s, as well known, Birkhoff and von Neumann found out quantum mechanics to follow a non-classical logic.[11] From physical evidence[12] the distributive law was taken by Birkhoff and von Neumann as the borderline between quantum logic and classical logic. However some following scholars axiomatized quantum logic by putting just the law of double negation to play such a role.[13] This fact reduces quantum mechanics to the above mentioned cases of scientific theories relying upon a non-classical logic. Moreover, this discovery proves that non-classical logic is not a side effect of just linguistic relevance in scientific theories, but it pertains to the core of the most advanced physical theories of the present century. It is clear why the discovery of non-classical logic in a physical theory by Birkhoff and von Neumann was not bypassed by physicists following classical logic: they indeed recognized the relevance of non-classical logic, since for the first time a DNS concerns the definition of a state of the system at issue. (Neither it can be confined to play a lateral role, outside the principles of the theory, nor can it be obscured by translating it by means of a misleading concept). On the contrary, in physical theories of the past DNSs concern statements that may be put aside when referring to the formal state of the system at issue. In chemistry the theory is expressed in linguistic terms, thus the DNSs may appear as a linguistic phenomenon only, not affecting the theoretical argumentation. However, we will show that, in the celebrated case of Avogadro's hypothesis, DNSs played a crucial role in the scientist's theoretical reasoning.

## 2. Double negated statements in the scientific tradition of chemistry

When scrutinizing chemical literature, we notice a common feature of the writings by even the early atomistic chemists; they state any crucial statement about atomism, for which they could not provide direct experimental evidence, as a double negated statement $\neg\neg A$, *i.e.* "it is not true that it is not [...]", or an equivalent version.

In fact, beyond reference to entities like air, water, earth, and fire, there was a great problem just at the starting point of their thinking, *i.e.* whether matter is divisible infinitely or not. It was a tenet of most of them that: "A mathematical divisibility *ad infinitum* does not apply to the matter of which the world is made".[14] The statement $A$: "A mathematical division *ad finitum* applies to matter" did not hold true during a long time, in which no one could decide by experimental means at which finite extent matter is divisible. In the lack of direct experimental evidence, chemists' dominant attitude was to take as a principle $\neg\neg A$.

Let us list some statements, whose dates range since the early times of such a theory. By starting his well-known dialogue Boyle suggests some basic statements; three out of four statements are double negated ones: "It not seems absurd [...]"; "It is not even impossible [...]"; "I will not deny peremptorily [...]".[15]

Lavoisier suggested a new method for chemical research. This method relied upon certain experimental rules about the weights of substances involved in a chemical reaction. As a result, the subsequent chemists knew well the distinction between assertions provable by experimental means and merely hypothetical assertions. As Lavoisier stressed: "J'espère que le lecteur voudra bien ne pas confondre ce que je donne pour des vérités de fait et d'expérience avec ce qui n'est encore qu'hypothétique".[16] Proust reiterates the same theme: "[…] to admit nothing provisionally, or beyond that which the facts set forth at present".[17]

However, the chemists following Lavoisier's method had to refer to the notion of elements, although they were aware of the lack of facts supporting their existence. Until the present century (let us recall Rutherford's α-beams and ultra-microscope), no experimental evidence for the existence of atoms had been provided. For this reason, when writing about the main problem of chemistry, Dalton says: "By elementary principles, or simple bodies, we mean such as have not been decomposed, but are found to enter into combination with other bodies. We do not know [by experimental means] that anyone of the bodies denominated elementary, is absolutely in-decomposable, but it ought to be called simple, till it can be analyzed."[18] Here the word 'simple' stands for $A$ in correspondence to the word 'in-decomposable', which stands for $\neg\neg A$. Indeed, if chemical atomism was just the research for the ultimate components of matter to be considered in chemical combinations, then in the early writings of atomistic chemists the word 'decomposable' naturally carries a negative meaning in the sense of 'not-ultimate' or 'not-simple'.

Dalton's reasoning by means of a double negated statement was the same as Lavoisier's on the same subject. Indeed, in defining the notion of elements twenty years earlier, Lavoisier writes: "Si […] nous attachons au nom d'éléments ou principes des corps l'idée du dernier terme auquel parvient l'analyse, toutes les substances que nous n'aurons pu décomposer par aucun moyen sont, pour nous, des éléments".[19] The first statement ("Si […] l'analyse") expresses a definition as a speculative idea, by means of exactly the positive statement $A$ of the double negated statement $\neg\neg A$ by which the quotation ends. Actually, he puts $\neg\neg A$ as a principle. Moreover, Lavoisier reports his most remarkable discovery in the following way: "Il est impossible douter de la composition et décomposition de l'eau".[20]

Some decades later, Wollaston writes: "[…] if it can be ascertained that any one body consists of particles no longer divisible, we then scarcely doubt that all other bodies are similarly consisted; and we may without hesitation conclude that those equivalent quantities, which we have learned to appreciate by proportionate numbers, do really express the relative weights of elementary atoms".[21]

The method of the subsequent chemists was fashioned by another of Lavoisier's statements: "Rien est créé, [nothing is not conserved] […] et un principe possible c'est que dans chaque opération la quantité de matière avant et après l'opération est la même".[22] Here the $\neg\neg A$ is stated first and then, as an improper derivation, $A$ is drawn. The derivation is an improper one because there is not enough experimental evidence for stating $A$; being aware of that, Lavoisier correctly qualifies the statement as a "possible" principle.

The way of arguing by means of DNSs became common among chemists; also, those who opposed the existence of atoms expressed their opinions by multiple negated statements. Kekulé wrote: "I do not hesitate in saying that, from a philosophical point of view, I do not believe in the actual existence of a-toms, taking the word in its literal signification of in-divisible particles".[23]

## 3. An historical case study: Amedeo Avogadro's hypothesis

The analysis of a specific case will show how such a way of reasoning can provide a methodological approach to some basic problems of science and, moreover, how much this approach was rooted in the minds of chemists.

Let us consider the first formulation of Amedeo Avogadro's well-known hypothesis. The hypothesis consists of two parts: the former one states the correspondence between gaseous volumes and the numbers of molecules, the latter one suggests the division of molecules in chemical reactions among gases.

Avogadro manifests these ideas for the first time in the *Essay d'une manière de determiner les masses relatives …* (1811).[24] In the following a detailed analysis of the first two sections out of the eight ones constituting the whole *Essay* is presented.[25]

### *Essay*: Section I

In the beginning of the *Essay*, Avogadro recalls that in 1809 Gay-Lussac had showed "que les combinaisons des gaz entre eux se font toujours selon des rapports très-simples en volume, et que lorsque le résultat de la combinaison est gazeux, son volume est aussi en rapport très-simples avec celui de ses composans".

Avogadro's aim is to give an account for these facts, by translating macroscopic objective data into hypotheses on the microscopic world. Because of the hypothetical nature and the lack of direct experimental evidence, he makes use of DNSs. He starts by suggesting a plain hypothesis: "Les rapports des quantités de substances dans les combinaisons ne paroissent[26] pouvoir dépendre que du nombre relatif des molécules qui se combinent, et de celui des molécules composées qui en résultent. Il faut donc admettre qu'il y a aussi des rapports très-simples entre les volumes des substances gazeuses, et le nombre des molécules simples ou composées qui les forme."

Afterwards, he sketches the first section of his hypothesis: "L'hypothèse qui se présent la première à cet égard, et qui paroît même la seule admissible, est de supposer que le nombre des molécules intégrantes dans les gaz quelconques, est toujours le même à volume égal, ou est toujours proportionnel aux volumes."

Avogadro is aware of the speculative nature of his guess; therefore, he offers logical evidence by means of an *ad absurdum* proof. Indeed, from the opposite assumption, *i.e.* that "le nombre des molécules contenues dans un volume donné fût différent pour le différens gaz", he draws a conclusion which is unacceptable, since it would prevent us from stating any connection between microscopic models and macroscopic experimental data: "il ne seroit guère possible de concevoir que la loi qui présideroit à la distance des molécules pût donner, en tout cas, des rapports aussi simples que les faits que nous venons de citer, nous obligent à admettre entre le volume et le nombre des molécules". As a matter of fact, he is following the methodological principle: it is impossible that the combining ratios among macroscopic bodies are not determined by the molecular ratios.

Let us note that Avogadro, in shaping his hypothesis, refers to Gay-Lussac's experimental results only, without using any physical hypothesis about the constitution of matter. After recognizing a relationship between gaseous volumes and the numbers of molecules, he has to qualify it by suggesting how molecules interact. To this end, Avogadro presents a physical model connecting molecules to his caloric theory.[27] His statements, being DNSs, show that the model is but a heuristic one. He says that any substance in the gaseous state, *i.e.* in the rarefied matter, attracts caloric in a specific way, but "sans que l'atmosphère [of caloric] formée par ce fluide ait plus d'étendue pour les unes que pour les autres [he does not say: 'it is equal for any substance'], et par conséquent, sans que la distance entre les molécules varie [not: 'the distance is the same'], ou, en d'autres termes, sans que le nombre des molécules contenues dans un volume donné soit lui-même différent [not: 'molecules number … is the same']".

He reports that Dalton maintains just the opposite sentence, *i.e.* that the amount of caloric which is attracted by every different gaseous substance is the same, yet it is differently condensed according to the specific affinity of each substance. According to Dalton, a variety of distances among molecules in different gases should exist. Avogadro disagrees: we are "dans l' [experimental] obscurité […] sur la manière dont cette attraction des molécules sur le calorique s'exerce". Truly, such an ignorance should lead us "à adopter une hypothèse mixte, qui feroit varier la distance des molécules et la quantité de calorique selon des lois inconnues". Instead the hypothesis "que nous venons de proposer", *i.e.* equal volumes of different gases contain the same number of molecules, has to be preferred to the previous one, because it is based "sur cette simplicité de rapport entre les volumes dans les combinaisons des gaz qui paroît ne pouvoir être autrement expliquée". Then, one page follows where Avogadro tries to apply his ideas to several gaseous substances.

## *Essay:* Section II

After the illustration of the correspondence between the volumes of gases and numbers of molecules, Avogadro has to give reasons to something more.

Let us illustrate the question by means of an example. Gay-Lussac observed that in the reaction of water formation the resulting volumes present the following ratios: 2 hydrogen volumes reacting with 1 oxygen volume to form 2 (gaseous) water volumes. Remarkably, the product volume of water is two times the volume of the reacting oxygen. As Avogadro puts it, such an experimental datum, "paroit d'abord s'opposer à l'admission de nôtre hypothèse à l'égard des corps composés […] mais il se présent assez naturellement un moyen d'expliquer les faits de ce genre conformément à notre hypothèse".

Indeed, in order to give an account for these data, Avogadro refers just to the correspondence between gaseous volumes and the numbers of molecules as proposed in the previous section. From both this assumption and Gay-Lussac's experimental data, he draws as a logical consequence the second part of his hypothesis, *i.e.* in chemical reactions the reacting molecules when changing into products may divide into parts. In the case of water formation, *e.g.*, one oxygen molecule divides up into two "molécules élémentaires", each one forming one water molecule; this hypothesis accounts for the observed volume ratios.

When illustrating his notion of molecules (*i.e.* the physical meaning of the latter hypothesis) he says they "ne sont pas formées d'une seule molécule élémentaire, mais résultent d'*un certain nombre* de ces molécules réunies en une seule par attraction".[27a] Avogadro tries to give a positive sentence; however the result is inaccurate, since no experimental evidence allowed him to define unambiguously how many "molécules élémentaires" constitute a gaseous molecule.]

Avogadro proves the second part of his hypothesis by *ad absurdum* arguments, too. The first proof is: "La possibilité de ce partage des molécules composées auroit pu être conjecturé même *a priori* car sans cela les molécules intégrantes des corps composés de plusieurs substances avec des nombres relatifs de molécules un peu considérables, deviendroient d'une masse excessive en comparaison des molécules des corps simples". His methodological principle is apparent: it is impossible that the masses of composed bodies are not comparable with the masses of simple bodies.

However, an *a priori* argument does not seem to be satisfactory. Hence Avogadro adds one more *ad absurdum* proof: "D'ailleurs une autre considérations paroît nous obliger d'admettre, dans quelques cas, le partage dont il s'agit; car comment pourroiton concevoir sans cela une véritable combinaison entre deux corps gazeux qui se réuniroient à volumes égaux, sans qu'il y eût condensation ainsi que cela a lieu dans la formation du gaz nitreux?"[28] The DNS is composed by the word 'sans' and by the rhetoric interrogation which waiting for a negative answer.

"Les molécules restant à la même distance à laquelle l'attraction mutuelle des molécules de chacun des deux gaz ne pouvoit s'exercer, on ne pourroit supposer qu'une nouvelle attraction eût eu lieu entre les molécules de l'un et celles de l'autre; [here, the methodological principle is the following: 'It is impossible that the kind of interaction among gaseous molecules drastically changes]; mais dans l'hypothèse [advanced by Avogadro] du partage, on voit bien que la combinaison réduit réellement deux molécules différentes à une seule, et qu'il y auroit contraction de tout le volume de l'un des gaz, si chaque molécule composée ne se divisoit pas en deux molécules de même nature." The last two phrases constitute a further *ad absurdum* proof: it is impossible to explain chemical reactions with the total volume of products being the same as that of the reactants, without resorting to a division of the reacting molecules.

Finally, let us address the question: which is the relation, if it exists, between DNSs and *ad absurdum* proofs, both throughout employed by Avogadro? It is clear that *ad absurdum* proofs constitute an instance of those apagogical or indirect proofs, which have been widely employed in science since the time of Euclid, as testified by the lively debate carried on until this century by philosophers, logicians, and scientists about the issue (Kant considered them as a watershed between philosophy and mathematics)[3]. The classical logical scheme of an *ad absurdum* proof is the following one:

$$(\neg A \to B) \ \to \ (\neg B \to \neg\neg A)$$

If $A$ is the problematic assertion, one starts with its negation ($\neg A$) and tries to derive a falsity or a contradiction $B$. Then one can infer $\neg\neg A$ from $\neg B$. Inference to $A$ is possible only through the law of double negation ($\neg\neg A \to A$).[29] Thus, in cases where the double negation law does not hold, as apparently in Avogadro's paper, the result of a *reductio ad absurdum* is a DNS.

# 4. Conclusions

Nowadays is commonly debated whether chemistry is independent of theoretical physics or not. Such a debate often refers to the deductive-nomological method of explanation illustrated for classical physics by Hempel and Oppenheim in 1948.[30] According to that method, given both a set of natural laws working as axioms – *explicans* – and the starting conditions, the particular – *explicandum* – is inferred. Such a method assumes both general laws and an inferring method (logical system). However, up to date, historians interested in supporting the specific theoretical role played by chemistry have pointed out that classical chemistry lacks those general laws,[31] which indeed constitute a typical feature of classical mechanics. It is unfortunate that the whole debate neglects the issue of chemists' logic, which truly makes apparent a more radical difference between the two theories.

We have shown that several scientific theories follow a specific logical path, which differs from the fully deductive path of classical mechanics. In the texts where these theories are originally formulated DNSs are apparent. Let us remark that in our century non-classical logic has eventually been acknowledged as a respectable one, since not the modest chemistry but the prestigious quantum mechanics has been shown to follow it.[12]

Today it is well known that the failure of the logical law '$\neg\neg A \rightarrow A$' constitutes the crucial difference of non-classical logic from classical logic. Here, on the basis of an analysis of original texts, we suggested that classical chemistry – as well as some of the theories belonging to physics and mathematics – has been shaped by authoritative scientists through a kind of logic which is at radical variance with that of the tradition of Newtonian mechanics. Therefore in our opinion chemistry radically differs from classical physics (Newtonian paradigm), not just because it is not built on general laws – indeed a DNS cannot play the usual role of an axiom, since the omission of the double negation law goes along with the omission of the law of the excluded middle –, as already stressed by other authors;[32] but essentially because it employs a different method, essentially based upon induction.

In particular, Avogadro formulated his hypothesis by means of this methodological approach, which was quite common among chemists; yet in Avogadro' writings, who was not an experimental chemist but just a speculative one, the approach is more apparent and it led him to more articulated arguments than before. Avogadro illustrated his hypothesis – a crucial point in the history of chemical atomism – by nothing more than DNSs and *ad absurdum* proofs. He actually made use of a double negated sentence whenever he had to express the result of *ad absurdum* reasoning. One may stress that, as chemists had to discover the 'indivisible particles' of matter without direct experimental evidence and yet they resulted successful when spectroscopy confirmed all their results, so Avogadro attempted to solve some problems about gases – today dealt by statistical and quantum mechanics –, and he was successful as well.

As already said, these two features – DNSs and non-classical logic – are inherently bound to one another. Together they suggest a specific method of organizing a greatly coherent scientific theory, as in the case of chemistry, around a problem that is implied by a crucial double negation. This kind of organization, which radically differs from the one of classical mechanics, will be the subject of a further paper.

## Notes and References

[1] J.B. Grize: "Logique" in: J. Piaget (ed.): *Logique et connaissance scientifique*, Encyclopédie de la Pléiade, Gallimard, Paris, 1970, pp. 135-288 (206-210); D. Prawitz, P.-E. Malmnaess: "A survey of some connections between classical, intuitionistic and minimal logic", in: A. Schmidt and H. Schuette (eds.): *Contributions to Mathematical Logic*, North-Holland, Amsterdam, 1968, pp. 215-229; D. Prawitz: "Meaning and Proof: the Conflict between Classical and Intuitionistic Logic", *Theoria*, **43** (1977), 6-39. In the following quotations, emphasis is added to mark double negations.

[2] G.W. Leibniz: Letter to Arnaud, 4-7-1686, in: C.I. Gerhardt (ed.), *Die Philosophischen Schriften von G. W. Leibniz*, Berlin 1875-1890 (repr. Hildesheim 1965), vol. II, p. 62.

[3] A recent historical review of the issue of indirect proofs (among which *ad absurdum proofs*) is included in P. Mancosu: *Philosophy of Mathematics and Mathematical Practice in the Seventh Century*, Oxford UP, 1996, in particular sect. 4.3.1. Mancosu reports that Kant maintained that: "When the grounds from which this or that knowledge has to be derived are too numerous or too deeply concealed, we try whether we may not arrive at the knowledge in question through its consequences." Therefore, "[…] the apagogical proof, […], while it can indeed yield certainty, cannot enable us to comprehend truth in its connection with the grounds of its possibility. The latter [proof] is therefore to be regarded rather as the last resort, than as a mode of procedure which satisfies all the requirements of the reason". Hence, it is only admissible in those sciences in which it is not possible "mistakenly to substitute what is subjective in our representations for what is objective, that is, for the knowledge of that which is in the object", in Kant's opinion mathematics (pp. 106-107).

[4] L. Carnot: "Réflexions sur l'infini mathématique", in: C.C. Gillispie: *Lazare Carnot Savant*, Princeton UP, 1971, pp. 169-262.

[5] N.I. Lobachevski: "Géométrie Imaginaire", *Crelle's Journal*, **17** (1837), 295-320.

[6] Quoted in: R. Dugas: *Histoire de la Mécanique*, Griffon, Neuchâtel, 1950, p. 219.

[7] L. Carnot: *Principes fondamentaux de l'équilibre et du mouvement*, Deterville, Paris, 1803, p. 49.

[8] L. Carnot: *Essai sur les machines en général*, Defay, Dijon, 1782, p. 44 (It. trans. by A. Drago and S.D. Manno: CUEN, Napoli, 1994).

[9] J.P. Joule: *The Scientific Papers*, London, 1884-87, vol. I, p. 322. It is remarkable that even S. Freud ("Die Verneinung", *Imago*, **11-3** (1925), 217-221) stresses that the whole transfer has to be conceived by negating the negated statements of the patient. See A. Drago: "Incommensurable scientific theories. The rejection of the double negation logical law", in: D. Costantini, M.D. Gavalotti (eds): *Nuovi problemi della logica e della filosofia della scienza*, CLUEB, Bologna, 1991, vol. I, pp. 195-202.

[10] The logical difficulty is shown by M. Jona: "What is energy?", *Physics Teacher*, **22** (1984), 6, by analyzing the current misleading definition of energy, as the capability to do work; actually energy is the capability to produce heat, otherwise heat would not be energy. Apparently the common definition assumes that "heat is work", rather than "it is not true heat is not work".

[11] G. Birkhoff, J. von Neumann: "The logic of quantum mechanics", *Annals of Mathematics*, **37** (1936), 23-43.

[12] For instance, the electron spin. For an introduction see R. I. G. Hughes: "Quantum logic", *Scientific American*, **245** (1981), 146-157.

[13] A. Fine: "Some conceptual problems of Quantum Theory", in: R.G. Colodny (ed.): *Paradigms and Paradoxes*, U. Pittsburgh P., 1972, pp. 3-31; P. Mittelstädt: *Philosophical Problems in Theoretical Physics*, Dordrecht, Reidel, 1978 (BSPS 18); R.W. Garden: *Modern Logic and Quantum Mechanics*, Hilger, Bristol, 1984; A. Drago: "Dualism and incompleteness of Quantum Mechanics", in: G. Garola, A. Rossi (eds.): *The Foundations of Quantum Mechanics*, Dordrecht, Kluwer, 1995, pp. 213-228.

[14] H. Guerlac: "Quantization in Chemistry", *Isis*, **52** (1961), 206. First remarks about double negations in chemistry are offered by A. Drago: "History of the relationships Chemistry-Mathematics", *Fresenius Journal of Analytical Chemistry,* **337** (1990), 220-224; *Erratum*, *ibid.,* **340** (1991), 787.

[15] R. Boyle: *The sceptical chymist*, London, 1661, first pages of part I.

[16] A.L. Lavoisier: *Traité élémentaire de chimie*, Paris, 1789, vol. I, p. 195. See also C. Meinel: "Early seventeenth-century atomism. Theory, epistemology and the insufficiency of experiment", *Isis*, **79** (1988), 68-103.

[17] L. Proust: "Lettre de M. Proust à M. D'Arcet", *Journal de Physique*, **59** (1804), 332.

[18] J. Dalton: *A new system of chemical Philosophy*, Pt. II, Manchester, 1810, pp. 221-222. See also: "When an element A has an affinity to another B, I see no mechanical reason why it should not take as many atoms of B as are presented to it", *Nicholson's J*., **29** (1811), 143-151. Here, and in the following quotations, our additions are included in square brackets.

[19] A.L. Lavoisier: *Œuvres de Lavoisier*, Paris, 1862-92, vol. 1, p. 7.

[20] A.L. Lavoisier: *Méthode de nomenclature chimique*, Paris, 1797, p. 298.

[21] W.H. Wollaston: "On the finite extent of the Atmosphere", *Philosophical Transactions*, (1822), 89-98.

[22] A.L. Lavoisier: *Œuvres de Lavoisier*, Paris, 1862-92, vol. 1, p. 15. It is well-known that Lavoisier's crucial innovation in chemistry method is what is commonly stated as 'matter is conserved'; yet the quotation makes apparent that Lavoisier deliberately stated this methodological principle by a DNS, whose corresponding affirmative sentence is dubious ("possible").

[23] A. Kekulé: "On the Existence of Chemical Atoms", *The Laboratory*, **1** (1867), 303-307, quoted in: R. Anschütz: *August Kekulé*, Berlin, 1929, vol. 2, p. 366.

[24] A. Avogadro, "Essay d'une manière de déterminer les masses relatives des molécules élémentaires des corps…", *Journal de physique, de chimie et d'histoire naturelle*, **73** (1811), 58-76. Reprinted in: M. Ciardi, "Introduction" to A. Avogadro: *Saggi e memorie sulla teoria atomica (1811-1838)*, Giunti, Firenze, 1995, pp. 39-65.

[25] For a detailed comparison of the historical interpretations of Avogadro's hypothesis see A. Drago and R. Oliva: "L'ipotesi di Avogadro. Storiografie a confronto e una nuova interpretazione", in: F. Calascibetta (ed.): *Atti del VII Convegno Nazionale di Storia e Fondamenti Chimica*, Rend. Acc. Scienze XL, **21**, pt. II, t. II, 1997, pp. 463-480.

[26] *Sic* in the original text, p. 39 of A. Avogadro: *Saggi…*, *op. cit.* Hereafter, other differences between Avogadro's French language and the modern French are not pointed out anymore.

[27] For Avogadro's caloric theory of gases, see R. Fox: *The caloric theory of gases*, Oxford, Oxford UP, 1971, pp. 196-226.

[27a] It is dubious if "une seule" represents a negation. If so, we have here a further instance of DNS.

[28] As M. Ciardi notes in A. Avogadro: *Saggi…*, *op. cit.*, p. 45, footnote 10, it is nitrogen monoxide, NO. In its formation reaction, 1 nitrogen volume reacts with 1 oxygen volume to give 2 oxide volumes. The total volume of products is equal to the volume of the reacting substances. This is the meaning of "a reaction without condensation". On the contrary, water formation reaction is an example of "a reaction with condensation"; in fact, the volume of the product is equal to 2/3 of the volume of its reacting substances.

[29] P. Mancosu: *Philosophy…*, *op. cit.*, p. 116.

[30] C.G. Hempel and P. Oppenheim: "Studies in the logic of explanation", *Philosophy of Science*, **15** (1948), 156.

[31] E.T. Caldin: "Theories and development of chemistry", *British Journal for the Philosophy of Science*, **10** (1959), 2091; G. Del Re, G. Villani, P. Severino, "Modelli di spiegazione scientifica e specificità della chimica", *Quad. Cooperativa Laboratorio*, **1** (1984) 25-32.

[32] G. Villani: "Specificità della chimica", *Philosophers in the Laboratory*, Acc. Naz. Science, Lettere e Arti di Modena, MUSIS, Roma, 1996, pp. 163-180; H. Laitko, W. Schmidt: "Tendenzen des chemischen Elementbegriffs", *Chemie in der Schule*, **15** (1968), 292.

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